Kennedy Baptist College

WAEP WA Exams Practice Paper C, 2016

Question/Answer booklet

MATHEMATICS APPLICATIONS UNITS 3 AND 4

Section One: Calculator-free

SOLUTIONS

Student number: In figures | | | | | | | | |

In words _____

Your name _____

Time allowed for this section

Reading time before commencing work: five minutes
Working time for section: fifty minutes

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer booklet
Formula sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	7	7	50	52	35
Section Two: Calculator-assumed	12	12	100	100	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Additional working space pages at the end of this Question/Answer booklet are for planning or continuing an answer. If you use these pages, indicate at the original answer, the page number it is planned/continued on and write the question number being planned/continued on the additional working space page.

5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you do not use pencil, except in diagrams.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

See next page

Section One: Calculator-free **35% (52 Marks)**

This section has **seven (7)** questions. Answer **all** questions. Write your answers in the spaces provided.

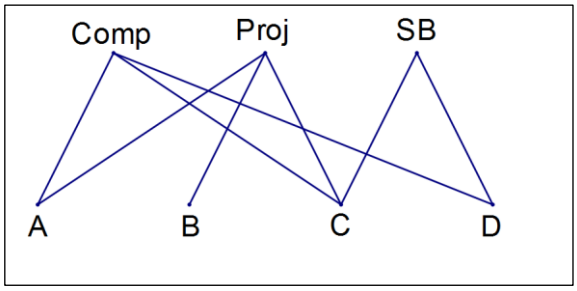
Working time for this section is 50 minutes.

Question 1 **(5 marks)**

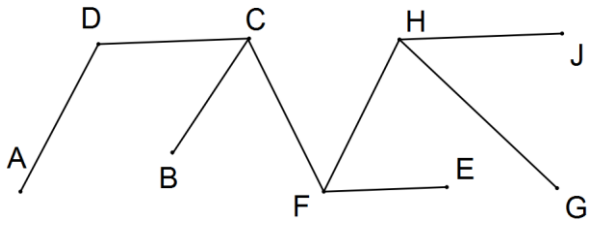
(a) The ticks in the table below indicate which facilities are available in each of four classrooms (A, B, C and D).

	A	B	C	D
Computer	✓		✓	✓
Projector	✓	✓	✓	
Smartboard			✓	✓

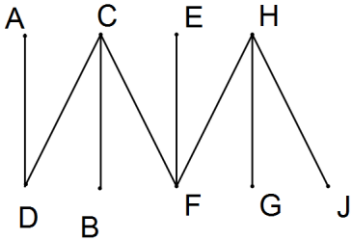
Show this information as a bipartite graph. (3 marks)



(b) State, with justification, whether or not the graph shown below is bipartite. (2 marks)



Yes. A, C, E and H form one set and B, D, F, G and J form the other, as shown below.



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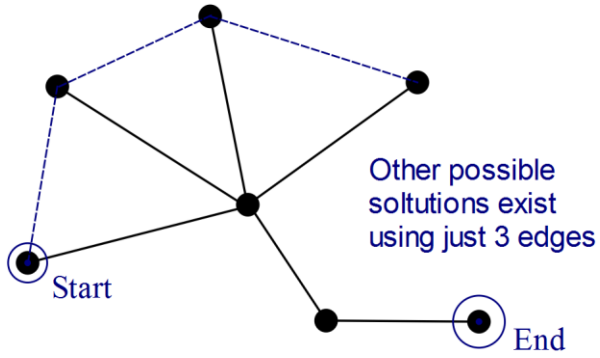
Question 2 **(8 marks)**

(a) The connected simple graph shown has 7 vertices.

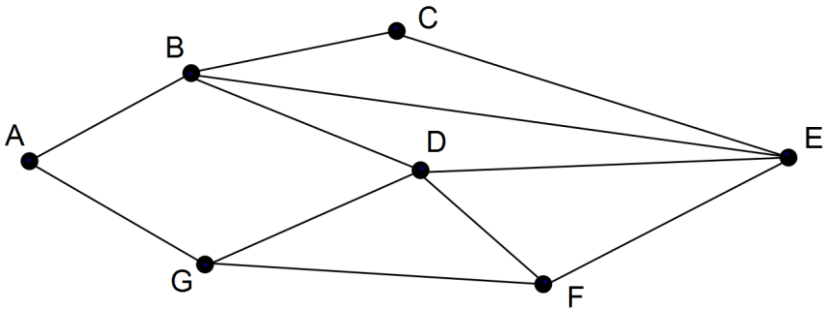


(i) Explain why this graph is a tree. (1 mark)

Trees are connected graphs that do not have a cycle.

(ii) Without adding any vertices, add the minimum number of edges to the above graph so that it contains a Hamiltonian path. Clearly indicate the start and end of the path. (3 marks)

(b) The graph shown below is semi-Eulerian.



(i) Clearly explain why the graph is semi-Eulerian. (2 marks)

The graph has just two odd vertices (G and F) and so a trail exists starting at one of these points and finishing at the other that visits every edge of the graph just once.

(ii) Classify the graph as Hamiltonian, semi-Hamiltonian or neither of these. Justify your classification. (2 marks)

Hamiltonian.

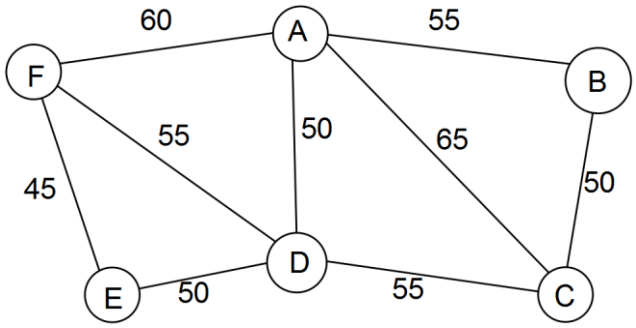
The graph contains a cycle that starts and finishes at one vertex (eg A) and visits all the other vertices just once.

Question 3 **(8 marks)**

A small park has nine pathways linking six entrances (A, B, C, D, E and F) together. The distances along pathways from one entrance to another are shown in the table below in metres.

	A	B	C	D	E	F
A	-	55	65	50	-	60
B	55	-	50	-	-	-
C	65	50	-	55	-	-
D	50	-	55	-	50	55
E	-	-	-	50	-	45
F	60	-	-	55	45	-

(a) Complete this weighted graph to represent the above information. (3 marks)



(b) Determine the shortest route from entrance E to entrance B, stating the vertices visited and the minimum distance. (2 marks)

(E) - D - C - (B)

50+55+50 = 155 metres.

(c) A dog-walker wanted to stroll along every pathway in the park, without walking along any pathway more than once. State whether this is possible and if it is, suggest a possible starting and finishing point. If it is not possible, briefly explain why not. (2 marks)

Yes.

Start at F and end at C (or vice-versa)

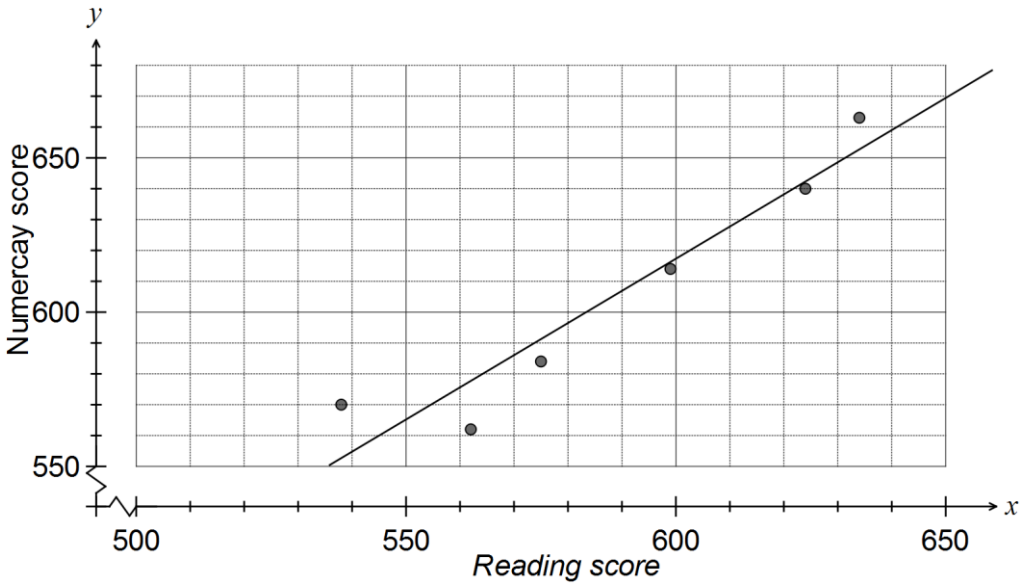
(d) Park authorities plan to create a second pathway from A to C. Explain whether this additional pathway will change your answer to (c). (1 mark)

Answer will change - walk is still possible, but now start at F and end at A.

See next page

Question 4 **(11 marks)**

Researchers looking for an association between reading and numeracy levels of students compared achievement at six schools using NAPLAN test results. The graph below shows the average reading and numeracy scores for the schools and the linear regression model $y = 1.042x - 8.055$ calculated from the data.

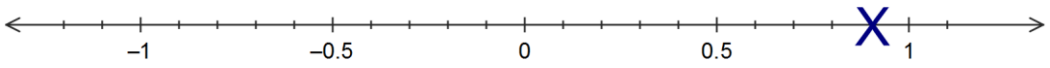


(a) Describe, in words, the association between the variables x (reading score) and y (numeracy score) for these schools in terms of direction and strength. (2 marks)

Strong, positive linear association.

(b) Consider the correlation coefficient r between the variables x and y.

(i) Place a cross on the scale below for your estimate of r. (1 mark)



Point must lie above 0.75 and below 0.95

(ii) Would you expect r to increase or decrease if the data point (538, 570) was removed from the data set? Explain your answer. (2 marks)

Increase.

The remaining five points lie much closer to a straight line.

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(c) A principal looking at the data concluded that because students at her school performed well at reading then they would perform well at numeracy. Comment on her reasoning. (1 mark)

Her statement is flawed - although an association can be observed, it does not mean that a causal relationship exists.

(d) One of the data points is at (562, 562). Would you predict that another school with a reading score of 562 will also have a numeracy score of 562? Justify your answer. (2 marks)

No.

The regression line should be used for a prediction rather than a data point and the line shows the prediction will be higher than 562.

(e) Another data points is at (634, 663). Write down a calculation to determine the residual for the linear regression model at this point, but do **not** evaluate it. (2 marks)

$(663) - (1.042 \times 634 - 8.055)$

(f) State one way the researchers could improve the reliability of their findings when investigating the association between reading and numeracy levels of students. (1 mark)

- Take a larger sample of schools

- Etc

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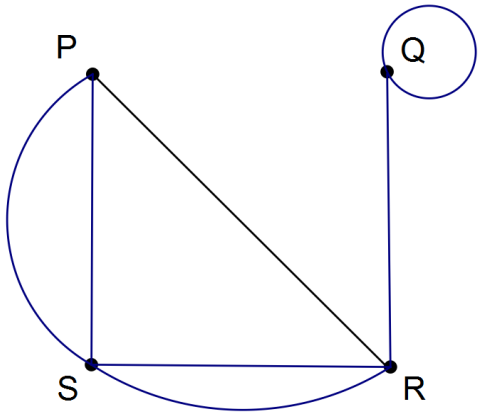
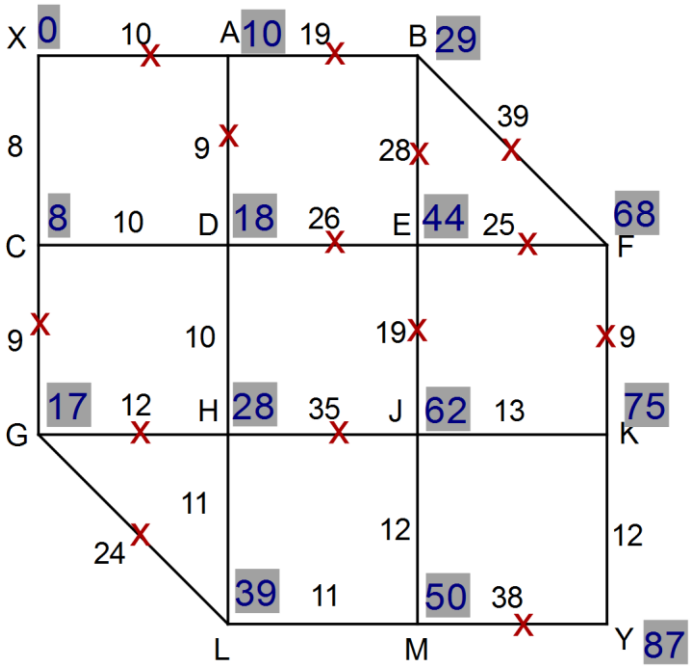
Question 5 **(6 marks)**

(a) Complete the non-directed graph below using the following adjacency matrix. (2 marks)

	P	Q	R	S
P	0	0	1	2
Q	0	1	1	0
R	1	1	0	2
S	2	0	2	0



(b) The number on each edge of the network shown below is the time, in minutes, required to travel between the vertices at the end of the edge. Determine the shortest time to travel from X to Y. List the vertices visited in order, from first to last. (4 marks)



Shortest time is 87 minutes.

Vertices: XCDHLMJKY

Question 6 **(8 marks)**

(a) A sequence is defined by $T_{n+1} = T_n + 10, \quad T_1 = 45$.

(i) Determine the next two terms of the sequence. (1 mark)

55, 65

(ii) State a rule for the n^{th} term of this sequence. (2 marks)

$T_n = 45 + (n-1)(10)$

(iii) Determine T_{45}. (1 mark)

$T_{45} = 45 + 44 \times 10 = 45 + 440 = 485$

(b) The first-order recurrence relation $T_{n+1} = aT_n + b$ was used with $T_1 = 3$ to calculate $T_2 = 4$ and $T_3 = 7$. Determine the values of a, b and T_5. (4 marks)

$3a + b = 4$

$4a + b = 7$

Subtract first from second:

$a = 3$

Substitute into first

$3(3) + b = 4$

$b = -5$

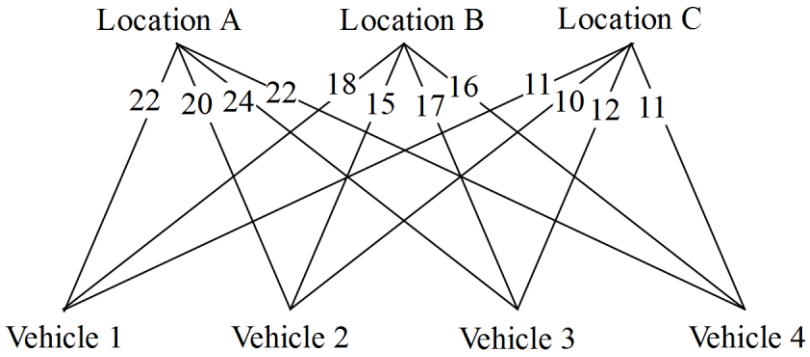
$T_4 = 3 \times 7 - 5 = 16$

$T_5 = 3 \times 16 - 5 = 43$

See next page

Question 7 **(6 marks)**

A transport company has packages to collect from three locations A, B and C, and has four vehicles that are available. The graph below shows the current distances of each vehicle from the locations in kilometres.



(a) Represent the information in the graph as a matrix. (1 mark)

$$\begin{bmatrix} 22 & 20 & 24 & 22 \\ 18 & 15 & 17 & 16 \\ 11 & 10 & 12 & 11 \end{bmatrix}$$

NB Rows and columns can be swapped.

(b) Show use of the Hungarian algorithm to determine which vehicle should collect which package in order that the total distance travelled by the vehicles is a minimum and state what this minimum distance is. (4 marks)

$$\begin{bmatrix} 22 & \underline{20} & 24 & 22 \\ 18 & \underline{15} & 17 & 16 \\ 11 & \underline{10} & 12 & 11 \\ 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 0 & 4 & 2 \\ 3 & 0 & 2 & 1 \\ \underline{1} & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & \underline{\underline{0}} & 3 & 1 \\ 2 & 0 & 1 & \underline{\underline{0}} \\ \underline{\underline{0}} & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

Add row to create square matrix
Reduce rows
Only 2 lines cover 0's so subtract/add 1
4 lines cover 0's so done

$11 + 20 + 16 = 47$ km

V1 collects from C, V2 collects from A and V4 collects from B for total of 47 km.

(c) If the initial distance of vehicle 3 from location A was reduced by 2 km, explain what effect, if any, this would have on your answer to (b). (1 mark)

No effect. When 24 is changed to 22, the algorithm would change 22 to 2 to 1, with no other changes to matrix.

End of questions

Additional working space

Question number: _____